### GeMA – XFEM Hydraulic Fracturing Examples



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### **Example set purpose**

This examples show:

- How to setup natural (initial) fractures on the model
- How to setup Injection Flow Rate in the model as a boundary condition
- How to setup boundary conditions on enriched degrees of freedom (displacement and porepressure)
- How to setup XFEM specific options
- Several different orchestration techniques for simultaneous or sequential hydraulic fracturing schemes with a geostatic step for external and internal forces equilibrium before the subsequent injection step



### The examples

 Example 1 presents a transient analysis of a single hydraulic fracture on a square plate to be compared with KGD analytical solution



 Example 2 and 3 present a transient analysis of a single hydraulic fracture entering another layer with symmetric and asymmetric stress contrast to be compared with Simonson and Fung analytical solutions, respectively.

Δσ=500 kPa	Δσ=1000	kPa	
∆σ=500 kPa	Δσ=500	kPa	



### The examples

• Example 4 and 5 present a transient analysis of three simultaneous and sequential hydraulic fractures, respectively with a homogeneous stress state.

 Example 6 presents a transient analysis of three simultaneous hydraulic fractures entering another layers with symmetric stress contrast varying fracture spacings of 7 and 20 m.



### The examples

On this presentation, the first example will present and analyze the complete model source. Other examples will present only relevant parts. The complete source for all models are available at the example files.

Although this examples try to explain all involved concepts and syntaxes, they are not a substitute for reading the GeMA tutorial and additional documentation.

As a convention, all the given examples will generate result files on the "out" directory.



# 1 – KGD ANALYTICAL MODEL IN K-VERTEX PROPAGATION REGIME



### **The Problem**

Injection on a square plate: It is considered a constant injection rate of a Newtonian, incompressible fluid in plane strain conditions in an infinite permeable or impermeable, homogeneous elastic medium.



Reference: "Propagation Regimes ofFluid-DrivenFracturesImpermeableRocks", Detournay, E.(2004)



## **Key Example Points**

- This example presents the basic structure for a hydraulic fracturing simulation in GeMA
- In particular it shows how to:
  - Structure a GeMA model
  - Define natural (initial) fractures in the model
  - Setup material values
  - Setup a boundary condition for the injection process
  - Set up the XFEM specific options
  - Get pressures at the injection point for post-processing analysis
  - Fill empty mesh for results visualization

Simulation file: KGD.lua



### Simulation file: KGD.lua

This file is the main simulation file. It stores a model description and loads the two auxiliary files storing the simulation model and the simulation solution. Splitting the simulation on those files is a convention to separate the description of what will be simulated (the model file) from the description of how it will be simulated (the solution file).





### Model file: State variable

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Next, the simulation state variable is defined. State variables are the nodal values calculated by the simulation and represent the model degrees of freedom. For a hydraulic fracturing simulation, state variables are:

```
-- State variables
StateVar{id = 'u', dim = 2, description = 'Displacements in the X and Y directions', unit = 'm',
          format = '8.4f', groupName = 'mechanic'}
StateVar{id = 'a', dim = 2, description = 'Enriched displacement in the X and Y directions', unit = 'm',
          format = '8.4f', groupName = 'mechanic'}
StateVar{id = 'P', description = 'Pore pressure degree-of-freedom', unit = 'kPa',
          format = '8.4f', groupName = 'hydraulic'}
StateVar{id = 'Pa', description = 'Enriched pore pressure degree-of-freedom', unit = 'kPa',
          format = '8.4f', groupName = 'hydraulic'}
StateVar{id = 'Pf', description = 'Fracture pressure degree-of-freedom', storage = 'both', unit = 'kPa',
          format = '8.4f', groupName = 'hydraulic'}
                                                                                    This state variable will have values for both
                State variables can be associated with different group names.
                                                                                    geometry nodes and ghost nodes (internal
                Those groups are used for prescribing different numeric tolerances
                                                                                    nodes created by the XFEM simulation)
                for different kinds of degrees of freedom at the solution file
```

## **Model file: Material properties**

For a hydraulic fracturing simulation, material properties are:

### PropertySet

```
= 'MatProp',
id
typeName = 'GemaPropertySet',
description = 'Material properties',
properties = {
  {id = 'E', description = 'Elasticity modulus', unit = 'kPa'},
  {id = 'nu', description = 'Poisson ratio'},
  {id = 'K', description = 'Hydraulic permeability', unit = 'm/s'},
  {id = 'qw', description = 'Specific weight of water', unit = 'kN/m3'},
  {id = 'Pht', description = 'Porosity'},
                                                                                 A map published by the Xfem
  {id = 'SPMax', description = 'Maximum principal stress', unit = 'kPa'},
                                                                                 plugin with its supported material
  {id = 'Gap', description = 'Initial gap opening', unit = 'm'},
                                                                                 model names
  {id = 'Ufw', description = 'Dynamic fluid viscosity', unit = 'kPa*s'},
  {id = 'Lkt', description = 'Leakoff at top', unit = 'm/(kPa*s)'},
  {id = 'Lkb', description = 'Leakoff at bottom', unit = 'm/(kPa*s)'},
  {id = 'material', description = 'Mechanical XFEM material type', constMap = constants.Xfem.materialModels},
},
values = {
  \{E = 17e+6, nu = 0.2, K = 9.8e-9, qw = 9.81, Pht = 0.2, SPMax = 1.25e3, \}
  Gap = 0.002, Ufw = 1e-7, Lkt = 2e-10, Lkb = 2e-10, material = 'poroElastic'},
                                                         A material type from constants.Xfem.materialModels
```



### Model file: Mesh

For a hydraulic fracturing simulation, Natural (Initial) fractures on the model are defined following the convention below:

### For one crack:

```
--Natural (Initial) fractures on the model
local meshInitialFractures = {
    --{{x1,y1}, {x2,y2}} initial and final crack points coordinates of Natural (Initial) fractures
    { {0.0, -0.5}, {0.0, 0.5}},
}
```

### For multiple cracks:



Initial and final crack points coordinates of natural (Initial) fractures must be coincide with an element border





## **Model file: Mesh**

#### Mesh

```
-- General mesh attributes
id = 'mesh',
typeName = 'Xfem.mesh',
description = 'Plate mesh discretization',
```

```
-- Mesh dimensions
```

coordinateDim = 2, coordinateUnit = 'cm',

-- State vars stored in this mesh (per node) stateVars = { 'u', 'a', 'P', 'Pa', 'Pf' },

```
-- Mesh node coordinates
nodeData = nodes,
```

```
-- Natural fractures
naturalFractures = meshInitialFractures,
```



The mesh name
 Xfem problems MUST use a specific mesh type provided by the Xfem plugin



The mesh dimension (2D) The unit in which node coordinates are given





Sets the table with node coordinates

	Associates	this mesh with	<b>Initial Fractures</b>
--	------------	----------------	--------------------------



## Model file: Mesh (continued)

### ... continued from previous slide

```
-- Element data
cellProperties = { 'MatProp', 'SecProp' },
                                                            Associates this mesh with property set MatProp and SecProp
                = elements,
cellData
                                                            Sets the table with element definitions
--IntegrationRules
elementRules = {
  \{quad4 = 2, tri3 = 3\}, --Rule set 1
                                                            Sets the available rule sets for xfem elements
  \{quad4 = 3, tri3 = 3\}, --Rule set 2
},
-- Node attributes
cellAttributes = {
  {id = 'pl', description = 'pressure loading applied at a border', dim = 2, unit = 'kPa'},
},
-- Boundary data
                                                            Sets the table with mesh border definitions
boundaryEdgeData = mesh edges,
```



## **Model file: Boundary conditions**

To complete the model file, boundary conditions for prescribing injection flow rates on plate borders are needed. Injection Flow Rate definition are in m3/s, negative signal means that flow is entering into the model. Here this condition is defined in a mesh node, then in the solution file is changed to the crack mouth node or injection point. This is done because initially no crack nodes are defined in the mesh, they only appear after the preprocessing, then crack nodes are added to the mesh.





## **Solution file: Numerical solver and Physics**

The first section of the solution file defines which numerical solver will be used to solve the equation system created by the XFEM method. On this example, we will use a direct matrix solver provided by the ArmadilloSolver plugin.

```
NumericalSolver {
    id = 'solver',
    typeName = 'ArmadilloSolver',
    description = 'Simple matrix solver',
}
The solver name
The plugin name used to create the numerical solver
```

Physics are the objects that provide the set of mathematical equations used to solve the simulation. For solving an injection problem, this example will use the Xfem plugin.

```
PhysicalMethod {
    id = 'HMCoupledXfem',
    typeName = 'Xfem.HydroMechanic',
    type = 'fem',

    mesh = 'mesh',
    ruleSet = 1, -- The integration rule set that will be used on the simulation
    boundaryConditions = {'bc1', 'bc3', 'bc4', 'bc5', 'bc6'},
    materials = {'poroElastic'},
    The physics name
    The physics with the mesh named 'mesh'
    The set of material types used by the simulation
}
```



## Solution file: solverOptions and xfemOptions

solverOptions and xfemOptions are the objects that provide specific options used to solve the simulation.





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<pre>local xfemOptions = {</pre>		
propagationCriteria	= 'MaxPS',	Maximum principal stress criterium for crack propagation
evaluationZone	<pre>= 'nonLocalQuad',</pre>	Create a quadrilateral region at the crack tip to compute the average
		maximum principal stress of the gauss points which are inside this region
weightFunction	= 'uniform',	Every gauss point inside the quadrilateral region used for MaxPS has the
		same weight value even though a point is nearer than other to the crack tip
geometricTol	= 1e-6,	Geometric tolerance to determine which gauss points are considered for
		Maximum principal stress calculation
geoStatic	= false,	False when no geoStatic step is performed
}		

Finally, the orchestration script, provided by the ProcessScript() Lua function, drives the simulation by calling the XFEM process to execute the simulation.

```
function ProcessScript()
   -- Just a definition of a parameter which represents the mesh object in the orchestration
   local m = modelData:mesh('mesh')
```

-- Just a definition of a parameter which represents the empty mesh object in the orchestration local em = modelData:mesh('emptyMesh')

-- Just a definition of a parameter to access the node value of the fracture pressure in the mesh "m" local pfAcc = m:nodeValueAccessor('Pf')

-- Create the solver model and execute the initial step in which preprocessing (crack nodes
-- are added to the mesh) is performed. In the xfem.init, the hydromechanical physic, the solver id,
-- solver options and xfemOptions are passed for the initial step
local solver = xfem.init({'HMCoupledXfem'}, 'solver', solverOptions, xfemOptions)

```
-- Changing node 1 in bc5 by the second ghost node to inject fluid flow rate
local bc = modelData:boundaryCondition('bc5')
bc:setNode(1, setMeshGhostFlag(2)) -- Second ghost node of the first crack
```



... continued from previous slide



```
-- The initial state of the analysis is saved io.addResultToMeshFile(file, 0.0)
```

--Definition of the file location which would contain the crack pressure along time local loadPf = io.open(translatePath('\$SIMULATIONDIR/out/KGDloadPf.txt'), "w+")

```
--Write the initial state of the three cracks pressure
loadPf:write("Inc\t Pf\n")
loadPf:write(0, " ", 0.0, "\n")
```



... continued from previous slide

```
--Definition of the parameters used in the orchestration
local dt = solverOptions.timeInitIncrement
local endt = solverOptions.timeMax
local nsteps = endt / dt
local dstep = 1 --FREQUENCE OF SAVING RESULTS ON EMPTY MESH
local cont = 1 --a counting parameter
local cont2 = dt*dstep --a counting parameter
for i=1, nsteps do
 --In the xfem.step, the solver parameters and time step are passed
 --Run transient analysis
 xfem.step(solver, dt)
 --After the analysis step finished, results are saved in the output file
 io.addResultToMeshFile(file, i*dt)
 --Get fracture pressure
 local Pfdata = pfAcc:value(setMeshGhostFlag(2))
 --write time, and fracture pressure
 loadPf:write(i*dt, " ", Pfdata, "\n")
```



### ... continued from previous slide

```
if (i*dt == cont2) then --If a different frequency of saving results is used then save results
--Save results in an empty mesh passing original mesh, empty mesh, solver parameters, displacements,
--pressures and fracture pressures. On the new mesh, Xfem sub-elements are transformed into regular
--mesh elements. Split fractures=true activates a postprocessing tool to visualize
--fracture aperture in the neutral file
xfem.copyToElementMesh(m, em, solver, {'u', 'a','P','Pa','Pf'}, {'S', 'E'},
{createMissing = true, splitFractures = true})
--save results of each step in the neutral file
```



# **Results:** KGD Analytical Model in K-vertex Propagation Regime (Detournay, 2004)



Good agreement is obtained between the profile of the crack mouth opening and the analytical solution. The pressure obtained from numerical simulations is always greater than the pressure obtained analytically because it neglects the hydro-mechanical coupled behavior of the surrounding porous medium, our model takes this coupling into account (Carrier & Granet, 2012; Mohammadnejad & Khoei, 2013).



# 2 – A SINGLE HYDRAULIC FRACTURE ENTERING ANOTHER LAYER WITH SYMMETRIC STRESS CONTRAST



## **The Problem**

• This example presents a transient analysis of a single hydraulic fracture entering another layer with symmetric stress contrast to be compared with Simonson analytical solution





## **Key Example Points**

- This example shows how to create a simple orchestration of a geostatic step which is intended to get equilibrium in the model for the initial state of a hydraulic fracturing simulation.
- This example shows the orchestration of the SIGINI subroutine which distributes the initial stress states in the model
- This example builds heavily on the previous one and only shows key difference points. Please refer to the simulation files for the complete source.

Simulation file: Simonson.lua



## **Solution File: SIGINI**

### This orchestration script distributes the initial stress state in the model.

assert(ir:numPoints() == 4) -- verify that elements contain 4 integration points



## **Solution File: SIGINI**

```
-- For each integration point
                                                          ... continued from previous slide
for j=1, ir:numPoints() do
  -- get integration point coordinates
  local ip, w = ir:integrationPoint(j)
  -- integration point in cartesian coordinates
 local coord = shp:naturalToCartesian(ip, Xnode)
  -- fill stress acording
 local Sv
  --These conditionals are set to define different stress state for three layers depending on depth
  if (coord(2) \ge 14) then
    Sv = -2000 --Vertical stress component
    Sh = -500 --Horizontal stress component
  end
  if (coord(2) < 14 and coord(2) > 7) then
   Sv = -2000 --Vertical stress component
    Sh = -500 --Horizontal stress component
  end
  if (coord(2) \le 7) then
   Sv = -2000 --Vertical stress component
    Sh = 0 --Horizontal stress component
  end
 --fill stress vector to pass it to the XFEM code
  local v = {Sh, Sv, Sh, 0} - stress vector
  accS:setValue(e, j, v) -- Passing element and gauss point indexes and stress vector to the XFEM code
end
```



### **Solution File: Geostatic Step**

-- Create the solver model and execute the Geostatic step in which preprocessing (crack nodes are -- added to the mesh) is NOT performed. In the xfem.init, the hydromechanical physic, the solver id, -- solver options and xfemOptions are passed for the Geostatic step

```
local solver2 = xfem.init({'HMCoupledXfem'}, 'solver', solverOptions, xfemOptions)
```

```
--this file is created to save the results of each step in one archive for postprocessing purposes
--such as analysis of variables along time
local file = io.prepareMeshFile(m, '$SIMULATIONDIR/out/Simonsontotal', 'nf',
{'u', 'a', 'P', 'Pa', 'Pf'}, {'S'}, {saveDisplacements=true})
```

```
--Definition of the parameters used in the orchestration
local dt = solverOptions.timeInitIncrement
local TimeFin = solverOptions.timeMax
local Time = dt
local nsteps = TimeFin / dt
for i=1, nsteps do
    --Run transient analysis
    --In the xfem.step, the solver parameters and time step are passed
    xfem.step(solver2,dt)
    --After the Geostatic step finished, results are saved in the output file
    io.addResultToMeshFile(file, i*dt)
end
```

--set current time to zero for the subsequent analysis (Injection Step) setCurrentTime(0.0)



### **RESULTS: Simonson's Analytical Solution**

According to Simonson's solution (1978), the injection pressure required for fracture penetration into the adjacent layers with symmetrical in-situ stress contrast is

$$P = \sigma_{res} + \frac{K_{1c}}{\sqrt{\pi L}} + \frac{2(\sigma_{barrier} - \sigma_{res})}{\pi} \cos^{-1} \left(\frac{h_{res}}{2L}\right)$$

$$h_{s} = 2L$$

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When the fracture penetrates into the adjacent layers where the in-situ horizontal stress is greater than that of the reservoir, the pressure required for fracture propagation increases (h > 7 m and h < -7 m).



## 3 – A SINGLE HYDRAULIC FRACTURE ENTERING ANOTHER LAYER WITH ASYMMETRIC STRESS CONTRAST



## **The Problem**

• This example presents a transient analysis of a single hydraulic fracture entering another layer with Asymmetric stress contrast to be compared with Fung analytical solution





### **RESULTS: Fung's Semianalytical Solution**

Fung et al. (1987) developed a semi-analytical procedure for computing the injection pressure required for fracture penetration into the adjacent layers with an arbitrary in-situ horizontal stress distribution.

$$K_{IC1} - K_{IC2} = \sqrt{\frac{h}{2\pi}} \sum_{i=1}^{n} \left\{ (\sigma_{i+1} - \sigma_i) \cdot \sqrt{1 - \left(\frac{2h_i - h}{h}\right)^2} \right\}$$

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$$K_{Im} = \sqrt{\frac{h}{2\pi}} \cdot \left\{ (P - \sigma_n)\pi + \sum_{i=1}^n (\sigma_{i+1} - \sigma_i) \cdot \left[ 2\sin^{-1}\sqrt{\frac{h_i}{h}} - (-1^m)\sqrt{1 - \left(\frac{2h_i - h}{h}\right)^2} \right] \right\}$$

The fracture penetrates farther into the lower layer due to lower in-situ stress in comparison with the upper layer.



 4 –TRANSIENT ANALYSIS OF THREE SIMULTANEOUS HYDRAULIC FRACTURES WITH A HOMOGENEOUS STRESS STATE IN A SINGLE LAYER AND FRACTURE **SPACINGS OF 7 AND 20 METERS.** 

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### **The Problem**

• This example presents a transient analysis of three simultaneous hydraulic fractures with a homogeneous stress state in a single layer and fracture spacings of 7 and 20 meters.







**Homogeneous stress state, Fracture Spacing =7 m** 



For a simultaneous scheme, stress shadowing effects of closely spaced clusters generates shorter and outward deviated side fractures and a straight longer fracture in the middle



**Homogeneous stress state, Fracture Spacing =20 m** 



As fracture spacing is increased (right), the stress shadowing effect decreases allowing all three fractures to propagate straight.





Due to the stress shadowing effect, fracture propagation occurs with a time lag between middle and side clusters. In contrast, a synchronized fracture propagation is observed when the spacing is enough to avoid stress interference between fractures.





As stress interference on fracture propagation increases, the breakdown pressure required to propagate increases as well. Consequently, a pressure difference can be observed between the side and middle fractures.

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The stress shadowing effect increases the fracture pressure required to propagate (left figure). As middle fracture propagates further than the side ones, less pressure needs to be accumulated due to the lower stress interference.



 5 –TRANSIENT ANALYSIS OF THREE **SEQUENTIAL HYDRAULIC FRACTURES WITH A** HOMOGENEOUS STRESS STATE IN **A SINGLE LAYER AND A FRACTURE SPACING OF 7 AND 20 METERS.** 



### **The Problem**

• This example presents a transient analysis of three sequential hydraulic fractures with a homogeneous stress state in a single layer and fracture spacings of 7 and 20 meters.







## **Multiple Sequential Hydraulic Fracturing**

### Homogeneous stress state



As fracture spacing is increased (from left figure to right), the stress shadowing effect decreases allowing all three fractures to propagate mostly straight.



## **Multiple Sequential Hydraulic Fracturing**



As fracture spacing is increased (right), no fracture closing is observed due to less stress shadowing effect and wider aperture is obtained for the last injected cluster

## **Multiple Sequential Hydraulic Fracturing**



As clusters are injected sequentially, the required pressure to propagate is increased due to the stress interference of the previous propagated fractures.



**6 – TRANSIENT ANALYSIS OF THREE** SIMULTANEOUS HYDRAULIC FRACTURES ENTERING ANOTHER LAYER WITH SYMMETRIC STRESS **CONTRAST AND FRACTURE SPACINGS OF 7 AND 20 METERS.** 



## **The Problem**

• This example presents a transient analysis of three simultaneous hydraulic fractures with a homogeneous stress state in a single layer and fracture spacings of 7 and 20 meters.





### Stress contrast between layers $\Delta \sigma = 500$ kPa



The stress shadowing effect and the stress contrast between adjacent layers increase the fracture pressure required to propagate